More often, it will result that fixed-range minimum-fuel trajectories fall into two categories. Those problems for which the range is sufficiently small that energy level Em is never attained [i.e., $-H_0 > -H_1(E_m)$], yield optimal trajectories consisting of a single powered climb and a maximum range glide. All other minimum-fuel trajectories consist of a climb path with $H_0 = H_1(E_m)$, a relaxed cruise of appropriate length at energy level E_m, and a maximum range glide.

Suboptimal Solutions

The optimum relaxed controller for the cruise segment achieves specific range $[-H_1(E_m)]^{-1}$. Rather than seeking controllers approximating the relaxed controller, one may select a suboptimal solution approximating the relaxed controller's performance.

Evaluation of aerodynamic data for the $F4$ yields a trivial difference between $-H_1(E_m)$ and the minimum value of $\sigma D/V$, as well as small differences in the energy levels at which $H_1(E)$ and $\sigma D/V$ are minimized. Thus, the energy state approximations support a classical cruise. In fact, investigations to date indicate that a cruise is superior to a full throttle climb and glide trajectory, whenever $\sigma D/V$ has a minimum within the flight envelope.

Secondly, from Fig. 1, for $0 \le -H_0 \le -H_1(E)$, point A does not vary significantly. Therefore, a Rutowski[4] minimum-fuel energy-climb path (i.e., $H_0 = 0$) is usually acceptable for the climb segment. Therefore, suboptimal trajectories using Rutowski climb paths, classical minimum cruise fuel cruises, and maximum range glides are also suggested.

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Sonic Boom Reflection Factors for Flight near the Threshold Mach Number

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WHEN an aircraft travels at a Mach number slightly greater than one, the sonic boom pressure wave which is generated by the aircraft is often unable to reach the ground due to atmospheric refraction. The maximum flight Mach number for which this will occur is called the threshold Mach number. The threshold Mach number depends upon the flight altitude and atmospheric conditions. For flight above 36,000 ft in the standard atmosphere the threshold Mach number is about 1.15. When an aircraft travels slightly faster than the threshold Mach number the wavefront of the sonic boom wave is nearly perpendicular to the ground where it strikes the ground. The magnitude of the sonic boom which is observed on the ground depends not only upon the amplitude of the incident pressure wave but also upon the manner in which the wave reflects from the ground. Because the sonic boom pressure wave is very weak it is usually assumed in sonic boom calculations that the amplitude of the pressure disturbance on the ground is, at most, twice the amplitude of the incoming wave; that is, the sonic boom reflection factor is assumed to have a maximum of two.

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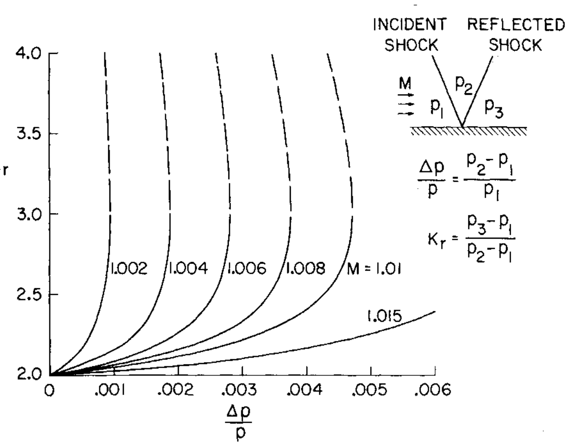

Fig. 1 Regular reflection of a very weak shock wave from a smooth surface.

The purpose of this Note is to consider the reflection of a very weak shock wave off a smooth surface, for the condition in which the incident shock wave is nearly perpendicular to the surface. It is found that in this situation the pressure rise Δp across the reflected shock can be up to twice the pressure rise across the incident shock, indicating that sonic boom reflection factors as large as three are possible for aircraft traveling near the threshold Mach number. Commercial transport aircraft which cruise slightly slower than the threshold Mach number are now under serious consideration for overland routes. Of course, if sonic cutoff does occur then the problem of reflection of the sonic boom wave does not arise. However, a change in the atmospheric conditions or in the elevation of the ground might cause the sonic boom pressure wave to reach the ground, and it is in this situation that a sonic boom reflection factor greater than two is possible.

The reflection of a very weak shock wave off a smooth surface, when the incident shock is nearly perpendicular to the surface, has been investigated using the oblique shock relations of Ref. 1 (Eqs. 128, 132, 139, and 150). Figure 1 shows the resulting relationship between the reflection factor (K_r), flow Mach number (M), and incident shock wave strength ($\Delta p/p$). It is seen that for values of $\Delta p/p$ typical of the sonic boom front shock and values of M close to one, the reflection factor can range between two and three. For any given $\Delta p/p$ and M there are two reflected shocks that are mathematically possible. However, the stronger solution, shown by dashed lines, does not occur in real life. Figure 1 is not intended to be used quantitatively in sonic boom calculations because $\Delta p/p$ cannot be estimated theoretically when the flight Mach number is very near the threshold Mach number. However, Fig. 1 does show that the sonic boom reflection factor should not be automatically assumed to be less than or equal to two, just because the sonic boom pressure wave is very weak. It is seen in Fig. 1 that for each Mach number there is a maximum $\Delta p/p$ for which a regular reflection is possible. If $\Delta p/p$ is larger than this maximum, then a Mach reflection occurs. The flow for this type of reflection is very complex due to the curvature of the shocks and the subsonic region downstream of the Mach stem. At present, theoretical methods are inadequate for predicting quantitative properties of the Mach reflection when the shocks are weak.

Reference

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